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Gauge theory incorporating automatically confined fields

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Abstract. If quarks and gluons are truly unobservable directly, this should be an inevitable consequence of the theory describing them. A formalism, within which 'internal fields' are automatically 'confined', is developed by considering a class of transformations of quantum mechanical states that have no explicit macroscopic manifestation. It transpires that one of the necessary consequences of this formalism is standard non-Abelian gauge theory, which arises in such a way that the results presented here seem to be consistent with accepted gauge-theoretic descriptions of sub-nuclear phenomena.

1. Introduction

The simplification of particle physics afforded by the concept of 'internal particles' (i.e. quarks and gluons) is now widely accepted (cf Close 1979). If, as is consistent with existing experimental evidence, these internal particles cannot manifest themselves directly, there must be a conceptual advantage in a formalism which ensures that 'confinement' is automatic and inescapable.

Coexisting 'external' and 'internal' fields are introduced here. Both species of field affect the dynamics of particles but only the external fields can appear explicitly in expressions for macroscopically measurable quantities. The formalism is derived from a class of transformations of quantum mechanical state vectors that have no observable manifestations. It transpires that the formalism is conventionally gauge-theoretic, and it seems that it must be possible to choose underlying Lie group structures for the internal and external fields to accord with currently accepted theories. The confinement of the internal field is inevitable.

2. Preliminaries

The usual quantum mechanical conventions

$$\langle\,|^* = |\,\rangle \qquad \text{and} \qquad \mathsf{P}^* = \mathsf{P} \tag{2.1}$$

are adopted, where upper case letters in bold sans serif type represent *quantum mechanical operators* and the asterisk denotes the *Hermitian transpose.* Particular *quantum mechanical states* are labelled with lower case script letters, e.g. $\langle \mathscr{s}|$.

Corresponding to any pair of states and any quantum mechanical operator, there is a *macroscopic quantity* identified by the same upper case letter as that which identifies the operator (but in script type):

$$\mathscr{P}(a, \ell) = \langle a|\mathsf{P}|\ell\rangle. \tag{2.2}$$

A Lorentz-covariant formalism is employed with lower case Greek suffixes identifying directions in space–time. The summation convention is invoked. Partial derivatives are denoted by ∂, e.g. ∂_μ, ∂^ν.

3. Macroscopically invisible transformations

What are here called *macroscopically invisible transformations* of the state vectors are characterised by unitary *transformation operators* written as upper case letters in italic type:

$$|\,\rangle' = S^*|\,\rangle \qquad \text{and} \qquad S^* = S^{-1}. \tag{3.1}$$

Corresponding quantum mechanical operators and macroscopic quantities are also identified by primes when they are transformed. The formalism developed in this paper is based on *macroscopic invisibility*, which is defined by

$$\mathscr{P}(a, b)' = \mathscr{P}(ae, b). \tag{3.2}$$

It follows from (2.1), (2.2) and (3.1) that

$$\mathbf{P}' = S^*\mathbf{P}S. \tag{3.3}$$

The dynamical behaviour of the state vectors is constrained by

$$\partial^\mu \mathscr{P}(a, b) = \langle a|\nabla^\mu \mathbf{P}|b\rangle \qquad \text{and} \qquad \partial^\mu \langle a|b\rangle \equiv 0 \tag{3.4}$$

where $(\partial^\mu \mathscr{P}(a, b))$ is required to be invariant under a macroscopically invisible transformation and $\nabla^\mu \mathbf{P}$ must be a quantum mechanical operator:

$$(\partial^\mu \mathscr{P}(a, b))' = \partial^\mu \mathscr{P}(a, b) \qquad \text{and} \qquad (\nabla^\mu \mathbf{P})^* = \nabla^\mu \mathbf{P}. \tag{3.5}$$

It is also required that physical constants and ordinary mathematical operations are invariant under macroscopically invisible transformations.

4. Gauge covariance

It follows from § 3 that $\nabla^\mu \mathbf{P}$ must be of the form

$$\nabla^\mu \mathbf{P} = \partial^\mu \mathbf{P} + \mathrm{i}q[\mathbf{W}^\mu, \mathbf{P}] \tag{4.1}$$

where $\mathrm{i} = \sqrt{-1}$, q is a real physical constant and $[\,\cdot\,,\,\cdot\,]$ denotes the *commutator*. The result of applying a macroscopically invisible transformation to $\mathbf{W}_\mu$ (it is not a quantum mechanical operator, even though $\mathbf{W}_\mu^* = \mathbf{W}_\mu$), which is called the *external vector potential operator*, is

$$\mathbf{W}_\mu' = S^*\mathbf{W}_\mu S - (\mathrm{i}/q)S^*\partial_\mu S. \tag{4.2}$$

Note that (4.1) and (4.2) accord with conventional gauge theory (cf Abers and Lee 1973, Goddard and Olive 1978)—hence the title of this section.

Straightforward manipulations show that

$$\nabla^\mu(\mathbf{QR}) = (\nabla^\mu \mathbf{Q})\mathbf{R} + \mathbf{Q}(\nabla^\mu \mathbf{R}) \tag{4.3}$$

for arbitrary quantum mechanical operators **Q** and **R**. Furthermore, it is found that (cf Atiyah 1979)

$$[\nabla^\mu, \nabla^\nu]\mathbf{P} = iq[\mathbf{Z}^{\mu\nu}, \mathbf{P}] \tag{4.4}$$

where the *external Yang–Mills tensor operator* $\mathbf{Z}^{\mu\nu}$ is defined by

$$\mathbf{Z}^{\mu\nu} = \nabla^\mu \mathbf{W}^\nu - \nabla^\nu \mathbf{W}^\mu - iq[\mathbf{W}^\mu, \mathbf{W}^\nu] = -\mathbf{Z}^{\nu\mu}. \tag{4.5}$$

5. Field tensor operator

Any field theory requires a conserved four-vector to act as the seat of the fields. If $\mathscr{J}^\mu$ is the macroscopic (i.e. classical) *current density* then it must satisfy

$$\partial_\mu \mathscr{J}^\mu \equiv 0. \tag{5.1}$$

Define $\mathbf{J}^\mu$, called the *current operator*, to be the quantum mechanical operator corresponding to $\mathscr{J}^\mu$. By analogy with (3.4) it follows from (5.1) that

$$\nabla_\mu \mathbf{J}^\mu \equiv 0. \tag{5.2}$$

Now introduce the *field tensor operator* $\mathbf{F}^{\mu\nu}$ defined by

$$\mathbf{F}^{\nu\mu} = -\mathbf{F}^{\mu\nu} \qquad \text{and} \qquad \nabla_\nu \mathbf{F}^{\mu\nu} = \mathbf{J}^\mu. \tag{5.3}$$

It follows from (5.2) and (4.4) that

$$[\mathbf{Z}_{\mu\nu}, \mathbf{F}^{\mu\nu}] = 0 \tag{5.4}$$

which is certainly satisfied by

$$\mathbf{F}^{\mu\nu} = \mathbf{Z}^{\mu\nu}. \tag{5.5}$$

This is another standard gauge-theoretic result.

6. Field operators

The main purpose of this paper is to develop a formalism within which the confinement of 'internal' fields/particles is automatic. To this end a particular class of quantum mechanical operators is introduced. The operator **R**, which is a typical member of this class, is expressed in the form

$$\mathbf{R} = \tilde{\Phi}\Theta + \tilde{\Theta}\Phi \tag{6.1}$$

where the upper case Greek letters Θ and Φ here represent what are called *field operators*, and $\tilde{\Theta}$ and $\tilde{\Phi}$ represent what are called *conjugate field operators*. Under a macroscopically invisible transformation, field operators and their conjugates transform according to the rules

$$\Theta' = T^{-1}\Theta S \qquad \text{and} \qquad \tilde{\Phi}' = S^{-1}\tilde{\Phi}T \tag{6.2}$$

where S belongs to the class of transformation operators introduced in (3.1), whereas T belongs to another class of transformation operators characterising what are here called *microscopically invisible transformations*. It follows from (6.1) and (6.2) that

$$\mathbf{R}' = \tilde{\Phi}'\Theta' + \tilde{\Theta}'\Phi' = S^{-1}\mathbf{R}S \tag{6.3}$$

which is seen on referring to (3.1) to be exactly equivalent to the transformation rule (3.3).

The 'internal' transformation, characterised by T, is invisible as far as the standard quantum mechanical operators are concerned.

On defining *right* and *left gauge-covariant derivatives*, $\overleftarrow{\nabla}^\mu$ and $\overrightarrow{\nabla}^\mu$ respectively, by

$$\tilde{\Phi}\overleftarrow{\nabla}^\mu = \partial^\mu\tilde{\Phi} + \mathrm{i}q\mathbf{W}^\mu\tilde{\Phi} - \mathrm{i}p\tilde{\Phi}\mathbf{V}^\mu \tag{6.4}$$

and

$$\overrightarrow{\nabla}^\mu\Theta = \partial^\mu\Theta + \mathrm{i}p\mathbf{V}^\mu\Theta - \mathrm{i}q\Theta\mathbf{W}^\mu \tag{6.5}$$

where p is a new real and positive constant, and $\mathbf{V}^\mu$ is called the *internal vector potential operator*, it is seen from (4.1) and (6.1) that

$$\nabla^\mu\mathbf{R} = (\tilde{\Phi}\overleftarrow{\nabla}^\mu)\Theta + \tilde{\Phi}(\overrightarrow{\nabla}^\mu\Theta) + (\tilde{\Theta}\overleftarrow{\nabla}^\mu)\Phi + \tilde{\Theta}(\overrightarrow{\nabla}^\mu\Phi) = \partial^\mu\mathbf{R} + \mathrm{i}q[\mathbf{W}^\mu, \mathbf{R}]. \tag{6.6}$$

The internal vector potential operator automatically vanishes from the gauge-covariant derivative of any quantum mechanical operator—i.e. it becomes 'microscopically invisible' or 'confined'!

Further manipulations of (6.4) and (6.5) show that

$$\tilde{\Phi}[\overleftarrow{\nabla}^\mu, \overleftarrow{\nabla}^\nu] = \mathrm{i}p\tilde{\Phi}\mathbf{Y}^{\mu\nu} - \mathrm{i}q\mathbf{Z}^{\mu\nu}\tilde{\Phi} \tag{6.7}$$

and

$$[\overrightarrow{\nabla}^\mu, \overrightarrow{\nabla}^\nu]\Theta = \mathrm{i}p\mathbf{Y}^{\mu\nu}\Theta - \mathrm{i}q\Theta\mathbf{Z}^{\mu\nu} \tag{6.8}$$

where the *internal Yang–Mills tensor operator* $\mathbf{Y}^{\mu\nu}$ is defined by

$$\mathbf{Y}^{\mu\nu} = \nabla^\mu\mathbf{V}^\nu - \nabla^\nu\mathbf{V}^\mu - \mathrm{i}q[\mathbf{V}^\mu, \mathbf{V}^\nu]. \tag{6.9}$$

The formulae (6.7) and (6.8), which appear to be new, demonstrate that there is no explicit cross coupling between internal and external Yang–Mills tensor operators.

For the field operators and their conjugates to be fully consistent with what has been developed in §§1–5, they must be further constrained such that $\mathbf{R}^* = \mathbf{R}$, in order that $\mathbf{R}$ can truly be a quantum mechanical operator. A convenient way of ensuring this is to require field operators to be, say, column vectors, and conjugate field operators to be row vectors, in an additional space—it accords with convention to call it *spin space*. Field operators and their conjugates of course remain tensors (or matrices) in the usual Hilbert space. Standard results are obtained by introducing quantities γ^μ that are vectors in space–time and tensors (square matrices) in spin space and which satisfy

$$\nabla^\nu\gamma^\mu \equiv 0 \tag{6.10}$$

and the conventional anti-commutation rules obeyed by the *Dirac matrices*—this is consistent with the standard spinor calculus (cf Brill and Wheeler 1957). When conjugate field operators are defined to be *Dirac adjoints*, e.g.

$$\tilde{\Phi} = \Phi^*\gamma^0, \tag{6.11}$$

then quantum mechanical operators, defined as $\mathbf{R}$ is by (6.1), are necessarily their own Hermitian transposes. Note also that

$$(\tilde{\Phi}\gamma^\mu\Theta)^* = \tilde{\Theta}\gamma^\mu\Phi. \tag{6.12}$$

7. Current operator

On invoking the notation developed in §6, the current operator introduced in §5 is conveniently expressed as

$$\mathbf{J}_\mu = \tilde{\Psi}\gamma_\mu\Psi = \mathbf{J}_\mu^* \tag{7.1}$$

where Ψ is a field operator whose characteristics have yet to be deduced. It follows from (6.4)–(6.6) and (6.10) that (5.2) is satisfied if the equation of motion for Ψ is

$$\gamma_\nu(\vec{\nabla}^\nu\Psi) = \mathrm{i}\mathbf{M}\Psi\mathbf{N} \tag{7.2}$$

where the operators $\mathbf{M}$ and $\mathbf{N}$ are such that

$$[\tilde{\Psi}\mathbf{M}\Psi, \mathbf{N}] = 0 \qquad \text{and} \qquad \mathbf{M}^* = \mathbf{M} \qquad \text{and} \qquad \mathbf{N}^* = \mathbf{N}. \tag{7.3}$$

Note that the equation conjugate to (7.2) is

$$(\tilde{\Psi}\bar{\nabla}^\nu)\gamma_\nu = -\mathrm{i}\mathbf{N}\tilde{\Psi}\mathbf{M}. \tag{7.4}$$

Inspection of (6.5) reveals that (7.2) is a generalisation of the *Dirac equation*, containing uncoupled internal and external vector potential operators. It is instructive to write down the second-order equation satisfied by Ψ, on the assumption that the operators $\mathbf{M}$ and $\mathbf{N}$ are constants under gauge-covariant differentiation, i.e.

$$\nabla^\mu\mathbf{M} = \nabla^\mu\mathbf{N} \equiv 0. \tag{7.5}$$

On introducing the conventional definition

$$\sigma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/\mathrm{i}2 \tag{7.6}$$

it is found that

$$\vec{\nabla}_\nu\vec{\nabla}^\nu\Psi - p\mathbf{Y}^{\mu\nu}\sigma_{\mu\nu}\Psi + q\sigma_{\mu\nu}\Psi\mathbf{Z}^{\mu\nu} + \mathbf{MM}\Psi\mathbf{NN} = 0 \tag{7.7}$$

which is seen to be a generalisation of the usual second-order equation (with interactions included). Remarkably enough there is no explicit cross coupling between the internal and external Yang–Mills tensor operators.

8. Automatic confinement

Provided there is negligible coupling to $\mathbf{V}^\mu$ by those matrix elements of Ψ that represent physical entities figuring in situations where the electromagnetic and weak forces are dominant, the formalism developed here accords with the accepted unification of these two forces. The conventional Lie group structure can be incorporated into $\mathbf{W}^\mu$ in the way explained by Iliopoulos (1977). Remember that gluon interactions can be neglected when comparing the predictions of weak/electromagnetic unification with experiment (cf Ryder 1977, p 113).

In general the operator Ψ must satisfy (7.2). So, the dynamics of the fermions described by Ψ are governed by interaction with the internal as well as the external vector potential operator. Although $\mathbf{V}^\mu$ cannot represent conventional gluons, there seems to be no good reason why its associated Lie algebra cannot be chosen to make it operationally equivalent.

Within the formalism presented in this paper, 'internal fermions' arise by the following reasoning. The argument begins by asserting that a family of external vector bosons (represented by $\mathbf{W}^\mu$) has been experimentally observed. There must be sources (i.e. four-currents) for these bosons. So, a current operator, such as the $\mathbf{J}^\mu$ defined by (7.1), is introduced. The macroscopic current density $\mathscr{J}^\mu$ must be conserved—i.e. (5.1) – implying that $\mathbf{J}^\mu$ must satisfy (5.2), which leads to (7.2). It is then asserted that experimental results are difficult to explain on the sole basis of external vector bosons, the evidence in fact suggesting the existence of internal vector bosons, represented by $\mathbf{V}^\mu$. However, there must be a current operator, $\tilde{\Phi}\gamma^\mu\Phi$ say, representing the sources of the internal bosons. Inevitably then, the field operator Φ must satisfy an equation equivalent to (7.2):

$$\gamma_\nu(\vec{\nabla}^\nu_{(1)}\Phi) = \mathrm{i}\mathbf{M}_{(1)}\Phi\mathbf{N}_{(1)} \tag{8.1}$$

where

$$[\tilde{\Phi}\mathbf{M}_{(1)}\Phi, \mathbf{N}_{(1)}] = 0, \qquad \mathbf{M}^*_{(1)} = \mathbf{M}_{(1)}, \qquad \mathbf{N}^*_{(1)} = N_{(1)}, \tag{8.2}$$

and, by analogy with (6.5),

$$\vec{\nabla}^\mu_{(1)}\Phi = \partial^\mu\Phi + \mathrm{i}o\mathbf{U}^\mu\Phi - \mathrm{i}p\Phi\mathbf{V}^\mu \tag{8.3}$$

where o is another real and positive constant, and $\mathbf{U}^\mu$ is a further vector potential operator (an 'internal–internal' one!).

Even though the fermions represented by the field operator Φ cannot actually be conventional quarks, there seems to be no good reason why the Lie algebra associated with $\mathbf{V}^\mu$ (note that this can be distinct from the Lie algebra associated with $\mathbf{W}^\mu$) cannot be chosen to make the formalism operationally equivalent to quantum chromodynamics.

Whatever the status of the internal vector potential $\mathbf{V}^\mu$ and the field operator Φ with respect to conventional gluons and quarks, they are inescapably confined because the formalism prevents them having any macroscopic manifestation. It is also worth noting that Φ and $\mathbf{V}^\mu$ exist everywhere that Ψ and $\mathbf{W}^\mu$ do, because their functional dependence is upon the same space–time.

An intriguing aspect of the formalism is that it provides automatically for an infinite sequence of families of vector bosons and fermions to 'nest' inside each other, like a kind of abstract Chinese puzzle. For instance, suppose that future experiments suggest the necessity for postulating further 'internal–internal' vector bosons, as represented by the operator $\mathbf{U}^\mu$ introduced in (8.3). It is perhaps worth recalling that there may even now be evidence for 'partons' possessing structure (cf Close 1979, §1.2). It follows that the bosons $\mathbf{U}^\mu$ must have sources, represented by say the current operator $\tilde{\Theta}\gamma^\mu\Theta$. The field operator Θ must then satisfy an equation analogous to (7.2) and (8.1). This leads to the possibility of introducing an 'internal–internal–internal' vector potential operator, which itself must have sources, and so on *ad infinitum*. Since the Lie algebras associated with each of the vector potential operators—$\mathbf{W}^\mu$, $\mathbf{V}^\mu$, $\mathbf{U}^\mu$, etc—can be treated separately, there need not necessarily be any dissatisfaction over not being able to construct the sort of 'master group' required for 'grand unification' (cf Harari 1978).

It has been shown here that the conventional gauge formalism is a consequence of the principle of macroscopic invisibility. But remember that the phase of $\mathscr{P}(a, b)$ cannot be observed. It might be worth attempting to find if any significantly different consequence follows from replacing $\mathscr{P}(a, b)$ on both sides of (3.2) by $|\mathscr{P}(a, b)|$.

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